NONSMOOTH PROBLEMS WITH APPLICATIONS IN MECHANICS

BOOK OF ABSTRACTS

MAY 19–23, 2025 | Olomouc, Czech Republic

Department of Mathematical Analysis and Applications of Mathematics
Faculty of Science, Palacký University Olomouc

Preface

Dear Colleagues,

Welcome to the international conference "Nonsmooth Problems with Applications in Mechanics 2025" (NOPAM 2025). This conference is a continuation of the successful event held in Będlewo in 2023. NOPAM 2025 aims to bring together researchers in the field of nonlinear mechanics. It serves as a forum for presenting new developments in theory, numerical mathematics, and optimization related to this area. We are pleased to announce a wide range of topics, including elasticity, elastoplasticity, fluid mechanics, porous media, and more. This year's scientific program also features nine invited lectures by renowned experts in the field. The social program includes a welcome drink, a gala dinner, and a guided sightseeing tour of Olomouc.

We wish you a productive and stimulating conference and a memorable stay in Olomouc.

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Abstracts

Invited lectures

Recent advances in the asymptotic analysis of contact problems for elastic shells

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Abstract: In recent years, considerable progress has been made in the mathematical analysis of elastic shells in contact mechanics (see for example [2, 3] and references therein). One of the most studied kinds of shells are the elastic elliptic membrane shells, an scenario where non trivial inextensional displacements do not exist (see [1]), i.e.:

$$V_0(\omega) := \{\boldsymbol{\eta} = (\eta_i) \in [H^1(\omega)]^3; \eta_i = 0 \text{ on } \partial\omega, \gamma_{\alpha\beta}(\boldsymbol{\eta}) = 0 \text{ in } \omega\} = \{\mathbf{0}\},$$

with $\omega \subset \mathbb{R}^2$ being the domain where it is defined a parametrization of the middle surface S of the shell, and $\gamma_{\alpha\beta}(\boldsymbol{\eta})$ denote the covariant components of the linearized change of metric tensor associated with a displacement field $\boldsymbol{\eta} = (\eta_i)$ of the surface S. Now, there is a wide range of situations where S cannot be assumed to be elliptic (just take for example a pipe), thus the need of availability of other shell models for contact problems. In this talk we will discuss recent advances in the asymptotic analysis of generalized membranes (where still $V_0(\omega) = \{\mathbf{0}\}$) and for the so-called flexural shells (where non trivial inextensional displacements do exist). We will analyze when these models are asymptotically equivalent to the corresponding Koiter's shells, as the thickness ε of the shell tends to zero. Further, it is common that frictional contact problems require taking into account tribological effects such as wear or adhesion. We will also briefly discuss recent progress in the asymptotic analysis of these kinds of contact problems for elastic elliptic membranes.

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On Optimal Design of Elastic Plates in a Dynamic Contact

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Abstract: We deal with an optimal design problem governed by initial-boundary value problems for a hyperbolic variational inequality describing deflections of elastic plates vibrating against an inner rigid obstacle. A variable thickness of a plate plays the role of a control variable. The set of admissible states for the design problem consists of solutions of a state problem gained as limits of the sequences of functions solving penalized problems. We assume the generalized penalized function $u \mapsto \eta^{-1}\beta\left(u - \frac{1}{2}e - \Phi\right)$ with a deflection u, a thickness function e and an obstacle function Φ. In the case of a differentiable function β and other more regular data it is possible to derive generalized optimality conditions starting with penalized problems. We apply the approach from [1], where a viscoelastic plate was considered. This case is an addition to the control problems solved in [2].

References

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Optimal control of systems governed by Stokes equations with non-smooth slip/leak conditions

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Abstract: In the first part of this contribution we present Stokes system equipped with several non-smooth slip and leak boundary conditions, recall their weak formulations and existence/uniqueness results. Special attention will be paid to the Stokes system with Tresca type leak condition which will be used in the subsequent part as the state problem. We derive its dual formulation in terms of three Lagrange multipliers releasing the divergence free condition, no-slip condition $u_t = 0$ on the leak part of the boundary, and regularizing the non-smooth leak term j, respectively. The algebraic counterpart of the dual formulation will be used as the state solver. The next part of the contribution deals with a class of optimal control problems in which the threshold leak bound g represents the control variable, Since the control-to-state mapping Φ is only Lipschitz continuous, sensitivity analysis being an integral part of any optimization problem has to be done using tools of nonsmooth analysis. We show how to get an "alternative" gradient information in the discrete case which will be then utilized in nonsmooth minimization methods. The last part of this contribution is devoted to computational aspects and several numerical experiments. We mention four approaches for solving this type of problems: a) fully nonsmooth, b) partially nonsmooth, c) fully smooth, d) gradient free global optimization approach. Our numerical experiments are based on b), c) and d) and the results will be compared.

Non-convex variational inequalities in contact mechanics

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Abstract: The rail traffic produces progressive deformations of the foundation, which may lead to a loss of stability. While the rail can be modelled with sufficient accuracy by the one-dimensional elastic or elastoplastic beam equation, the behavior of the foundation is more complex and includes irreversible structure changes such as decrease of the void ratio and degradation of the material. We describe the full problem by a partial differential equation coupled with a nonconvex variational inequality and prove that it admits a unique solution.

Weak solutions for compressible viscoelastic fluid models in three space dimensions

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Abstract: We discuss global in time existence of weak solutions to compressible viscoelastic fluid models in three space dimensions. The first result concerns the situation with corrotational derivative in the extra stress tensor. Then, assuming additionally that the extra stress tensor has a particularly simple structure, the existence of weak solutions can be shown even in the situation when the stress diffusion is neglected which is often the case in applications.

The second result concerns Oldroyd-B type of model. It is known that in three space dimensions the Newtonian structure for the viscous part of the stress tensor is not enough to ensure the existence of weak solutions for arbitrarily large data. However, assuming the stress tensor of the power-law type it is possible to close the estimates and construct solutions provided the extra stress diffusion is present and the model of the viscous stress tensor provides bounded velocity divergence.

The Damped Normal Compliance contact condition and frictional slip waves

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Abstract: The condition takes into account energy dissipation during the contact process. We describe the three cases of impact of a rigid body, a rod or a beam on a DNC obstacle. These works deal with the models, analysis and computer simulations of the contact processes. Then, we describe two results on the initiation of sliding in systems with spring-mass-damper when friction is taken into account, of two and three masses. Finally, we describe a few unresolved issues in these models that are of interest for further research.

A Convergence Criterion for History-dependent Variational Inequalities

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Abstract: We consider a variational inequality in a reflexive Banach space X, governed by a history-dependent operator. The existence of a unique solution to the inequality is proved by using a fixed point argument. Based on the fixed point structure of the problem, we provide necessary and sufficient conditions which guarantee the uniform convergence of a sequence of functions to the solution. We exploit this result both in the study of a penalty method and the well-posedness analysis of the problem. Moreover, we present its application in the study of a mathematical model which describes the equilibrium of an elastic body in contact with a rigid-plastic foundation. The contact is frictionless and the hardening of the foundation is taken into account. We use our abstract results to obtain the continuous dependence of the solution with respect to various data and parameters. Finally, we use a finite element scheme to approximate the problem, implement it on the computer and provide numerical simulations which validate the theoretical convergence results.

Contributed talks

From Sharkovsky theorem to topological entropy for multivalued maps

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Abstract: Starting from the celebrated Sharkovsky cycle coexistence theorem, we will show that its application to differential equations leads to an "empty" theorem. In the lack of uniqueness, such an application is possible, provided we have to our disposal a multivalued version of the Sharkovsky theorem. In order to calculate topological entropy of given differential equations or inclusions, via the associated Poincaré translation operators along their trajectories, we need a new definition of topological entropy for multivalued maps. Its positive value can be achieved by means of the Bowen-Franks type theorem (based on the mentioned Sharkovsky-like extension) in terms of subharmonic periodic solutions whose periods differ from the power of two. On the other hand, despite a consistence with a single-valued case, there exist definitions of topological entropy equal to zero.

Subharmonic vibrations of microbubbles in the cloud driven by ultrasound

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Abstract: This work investigates the dynamics of microbubbles driven by ultrasound. Due to the nonlinearities inherent in the governing equations, advanced numerical methods are required for accurate analysis. Instead of using commercial software, dedicated algorithms were developed and applied to solve these model equations. Dynamical simulations were conducted for systems containing 10, 20, and 100 bubbles of either uniform or random sizes, randomly distributed in space. The temporal evolution of bubble sizes, emissivity, and inter-bubble interactions was analysed. The results demonstrate that even small variations in bubble polydispersity can significantly influence the components of the scattered pressure spectrum. Increasing the dispersion of bubble radii led to the emergence of half-subharmonics across the entire range of driving frequencies. In contrast, for non-interacting bubbles of uniform size, half-frequency subharmonics appeared only at a single excitation frequency corresponding to twice the resonance frequency of an individual bubble. A detailed stability analysis reveals that phenomena previously interpreted as chaotic behavior in bifurcation diagrams are, in fact, artifacts of the numerical methods employed.

Elastic Body Simulation under Real-time Regime

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Abstract: We present an approach to elastic bodies simulation in a real-time physics engine software. In the talk we focus on the application of the St. Venant-Kirchhoff stress tensor in the deformable body dynamics simulation. Then we proceed to the implementation of the Finite Element Method in discretisation of an elastic body in three-dimensional space and explain how to numerically solve a specific system of partial differential equations. In addition, we examine a couple of setbacks that arise in the process of the simulation design such as numerical instability, linearisation or the boundary condition definition. In order to better understand these issues, we explain what the "real-time regime" is and how it impacts the approach to the software design. Moreover, we move on to the interactions between rigid and elastic bodies as a variety of these problems often undermines the reliability of real-time simulations. The difficulty in modelling such contacts is caused by frequently occurring nonsmooth relations. Thus, a couple of cases are shown in an interactive mode during the presentation. In the end we discuss the design of BartaEngine `https://github.com/Bartanakin/BartaEngine` – software that exhibits the most important features of a real-time physics engine and is written in the modern C++ language. Our talk is partly based on Jernej Barbic's research [1] who is a leading scientist in computer graphics.

References

[1] Sifakis, E.,& Barbič, J., *Finite element Method Simulation of 3D deformable solids*, Morgan & Claypool Publishers 2016.

Global Subgradient Method for hemivariational inequalities

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Abstract: In this talk, we employ a global aggregate subgradient method for the numerical solution of hemivariational inequality problems arising in contact mechanics. The method integrates a global search procedure to identify starting points for a local minimization algorithm. The algorithm consists of two types of steps: null steps and serious steps. In each null step, only two subgradients are utilized: the aggregate subgradient and the subgradient computed at the current iteration point, which together determine the search direction. Furthermore, we compare the performance of the proposed method with selected solvers using a representative contact mechanics problem as a case study.

Vanishing viscosity method for a noncoercive hyperbolic differential hemivariational inequality

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Abstract: We consider a coupled system consisted of a second order hemivariational inequality (HVI) and an ordinary differential equation (ODE). Our goal is to provide an existence of a solution for the above problem. It has been done recently in [1] by means of the Rothe method. Now we apply an alternative technique based on a vanishing viscosity method. The idea comes from [2], where the vanishing viscosity method was used in the study of hyperbolic HVI without additional ODE. We also discuss applications of the theoretical result in contact problems and formulate some open questions.

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[2] S. Migorski, A. Ochal, *Vanishing viscosity for hemivariational inequalities modeling dynamic problems in elasticity*, Nonlinear Analysis 66 (2007) 1840–1852.

Existence and regularity results for variable exponent systems

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Abstract: We are concerned with the study of variable exponent systems involving generalized operators of Leray-Lions type. More precisely, we provide existence and regularity results. At the same time, we aim to motivate our preference for this kind of operators by presenting various examples of systems that represent particular cases of the general class of systems treated by us.

Reliable Lower Eigenvalue Bounds for Beam Buckling Problem

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Abstract: In the stability analysis of elastic structures, accurately estimating the critical buckling load is essential for safe and efficient design. This problem is commonly formulated as an eigenvalue problem, where the smallest eigenvalue corresponds to the critical load. However, standard numerical methods often provide upper bounds on these eigenvalues, but not lower bounds. This limits the reliability and practical applicability of such estimates. This contribution presents a framework for deriving guaranteed and computable lower bounds for eigenvalues associated with buckling problem for both linear Euler-Bernoulli model and nonlinear Gao beam model. We demonstrate how these lower bounds can be computed alongside standard numerical approximations, particularly those based on finite element discretization, resulting in two-sided estimates for the critical buckling load. We illustrate the approach with numerical examples and discuss its potential extensions.

A unified approach for $p-$Laplacian differential inclusions depending on a parameter

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Abstract: A unified approach was developed to investigate the existence of at least one smooth nontrivial solution for p-Laplacian differential inclusions with upper/lower semicontinuos set-valued righthand side and depending on a positive parameter. The main idea was to find a solution of the given inclusion as a solution of an auxiliary differential inclusion associated with the generalized gradient (in Clarke's sense) of a primitive of a selection of the multivalued nonlinearity. Since we also dealt with discontinuous selections, variational methods for non smooth functionals provided the right tools to achieve our goals.

References

[1] G. Bonanno, P. Candito, F. Cianciaruso, P. Pietramala, *A unified approach for p - Laplacian differential inclusions depending on a parameter* Discrete Contin. Dyn. Syst. Ser. S 18 (2025), no. 2, 553–565.

Discontinuous behavior of the thermal dissipation in a differentially heated cavity with conjugate heat transfer

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Abstract: Conjugate heat transfer is encountered in most real-world applications and has a major impact on the physics of a wide range of industries, including heat exchangers, chemical reactors, solar collectors, and electronics. In Computational Fluid Dynamics (CFD), accurately predicting temperature fluctuations in fluid flow is both challenging and crucial. Indeed, these fluctuations can cause variations in adjacent solid walls, leading to cyclic thermal stresses and potential fatigue cracking.
Several studies have investigated conjugate heat transfer in turbulent forced convection. However, to the best of the author's knowledge, no reference data exist for turbulent natural convection, despite the common occurrence of such flows. This study takes a step toward addressing this gap in the literature.
Thermal dissipation, involving the derivatives of the temperature fluctuations, exhibits a nonsmooth behavior at the fluid-solid interface, posing a major challenge for accurate physical modeling. The present study explores the coupling between flow dynamics and thermal penetration in the solid through high-resolution simulations and advanced numerical methods, aiming to analyze the underlying physics and provide datasets for the modeling community.

Stochastic Models of Damaged Interfaces: A Numerical Study

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Abstract: Predicting damage in structures, particularly in composite structures, is essential to reduce the risk of failure. In this presentation, numerical strategies are presented to simulate the solid/solid interface behaviour, where a special attention will be paid to damage. Damage can occur at the scale of the volume of the material or at the scale of the interfaces between the materials that make up the structure. Damage is a highly complex phenomenon. In the work we are proposing, we consider the effect of the growth of micro-cracks at the micro-scale. Since this degradation is not totally predictable, uncertainties are introduced using the Itô integral. Secondly, two interface models are proposed. For this purpose, a technique of asymptotic developments is realised, allowing interface models to be derived from previous volume models. Finally, some numerical results obtained for academic examples will be presented, in order to understand the behaviour of the models and the influence of material parameters.

Multiple positive solutions for quasilinear nonlocal problem via topological, variational and set-valued methods

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Abstract: Our goal is to prove existence and multiplicity of positive solutions for the following problem

$$\begin{cases} -a\left(\displaystyle\int_\Omega u^q dx\right)\Delta_p u = f(x,u) & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{P}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary, and the driven operator is the classical p-Laplacian, i.e., $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)\ \forall u \in W_0^{1,p}(\Omega)$. Moreover, we assume that a is a continuous functions, possible sign changing and $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function. The main novelty in our result is the absence of monotonicity conditions on the reaction term. Our result provides the existence of pairs of positive solutions for problem (P) ordered in L^q-norm. In particular, we combine sub-super solution and variational methods with truncation techniques to obtain positive solutions for a counterpart of the problem (P) with "frozen" non-local term. Next, to come back to solutions of our original problem, first, we prove the existence of the smallest solution for the "frozen" problem through a suitable set-valued map. Finally, we define a one-dimensional fixed point problem that leads to the existence, multiplicity and ordering of solutions of (P). We highlight that these nonlocal problems were introduced in 1945 by Carrier to describe the beams' deflection, but, in recent years, a wide interest in these equations has led many authors to obtain models in biology and engineering.

References

[1] P. Candito, G. Failla, L. Ganiński, R. Livrea *Multiple positive solutions for quasilinear nonlocal problem via topological, variational and set-valued methods*, (Preprint).

On the existence and uniqueness for mixed local-nonlocal Dirichlet boundary value problems

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Abstract: By applying the method of monotone operators and techniques related to the method of sub-super solutions, we examine the existence and uniqueness of mixed local-nonlocal Dirichlet boundary value problems with convection depending on the gradient. We also consider the continuous dependence on parameters for such problems. To this end, we derive a parameter-dependent surjectivity theorem. Applications to problems involving an unbounded weight are also provided.

Nonlocal p-Laplacian problem and Carrier's double phase problem

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Abstract: We obtain nonexistence, existence and multiplicity results for a nonlocal p-Laplacian as well as for a nonhomogeneous operator with unbalanced growth problem. To tackle these challenges, we employ a combination of analytical techniques, including the sub-super solution method, variational and truncation approaches, and set-valued analysis. Furthermore, we examine a one-dimensional fixed-point problem.

Regularity lost: the fundamental limitations and constraint qualifications in the problems of elastoplasticity

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Abstract: The connection between plasticity phenomenon in mechanics and the sweeping process was noticed since the very discovery of the sweeping process as a mathematical problem. Using the ideas of J.-J. Moreau, we describe abstract frameworks for elastoplasticity in terms of adjoint linear operators and convert them to equivalent formulations in terms of differential inclusions with normal cones.
However, with the seemingly simplest constitutive law of perfect plasticty it is, in general, impossible to find the strain rate as a L_2-function. Although some examples with such phenomenon are already known, we provide an example where the lack of a function-valued strain is caused by a displacement loading. This loss of regularity can be explained by the lack of additivity of the normal cones and the failure of Slater's constraint qualification.
Various constraint qualifications can help to establish the additivity of the normal cones and show well-posedness of the similar models, such as spatially discrete models and plasticty with asymptotically linear hardening.
Preprint: https://arxiv.org/abs/2412.13068

A Stability Result for Variational Inequalities Applied to Unilateral Problems in Mechanics

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Abstract: This talk presents a new stability result for classes of variational inequalities and of hemivariational inequalities. We admit perturbations not only in the right hand side, but also in the monotone operator, the locally Lipschitz function, and in the constraint set. While Mosco convergence is the standard choice of set convergence to treat approximation and stability of the constraint set, see e.g. [1, 2], here we employ Hausdorff set convergence.This applies to various nonsmooth boundary value problems in mechanics, in particular to unilateral contact problems.

References

[1] O. Chadli, J. Gwinner, N. Ovcharova, *On semicoercive variational-hemivariational inequalities - existence, approximation, and regularization*, Vietnam J. Math. 46 (2018) 329–342.

[2] Liang Lu, Lijie Li, M. Sofonea, *A generalized penalty method for differential variational-hemivariational inequalities*, Acta Math. Sci. Ser. B (Engl. Ed.) 42 (2022) 247–264.

Flow degeneracy in a fractured porous medium

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Abstract: Numerous applications of subsurface engineering involve injection and extraction of fluids. Examples include geothermal energy extraction, nuclear waste storage, carbon sequestration, petroleum engineering applications, and energy storage. These anthropogenic activities involve a complex set of processes involving flow, thermal, chemical reactions, and mechanical effects all possibly coupled to each other. These complex sets of processes interact with the complex geology that involves ubiquitous fractures and faults. The network of fractures forms the primary conduit of flow and transport. The fractured medium is generally anisotropic, heterogeneous, and has substantially discontinuous material properties spanning several orders of magnitude.

We consider degenerate models arising out of an evolving porous media including fractures. When the porosity becomes smaller and reaches zero, the permeability also degenerates including in the fractures. We consider the situation when the porosity as well as the permeability in the porous matrix and in the fracture may vanish on sets of positive measure. We consider a mixed dimensional version of the Darcy flow model that couples the fluid flow on a fractured surface to the flow in the matrix. As porosity vanishes, the pressure becomes uncontrolled due to the absence of fluid phase. By introducing porosity-weighted scaled variables for pressure and velocity, we derive scaled fields that are mathematically well-defined and establish their correspondence with the physical fields. We introduce weighted Hilbert space for both matrix and fractured variables and show the well-posedness of mixed dimensional problem in this weighted setting. For discretization, we consider the lowest order Raviart - Thomas mixed finite element spaces for the Darcy velocity and piece-wise constant pressure on simplexes. Numerical results demonstrate optimal convergence rates.

This is a joint work with Sundus Iqbal (Bergen) and Omar Duran (Stanford).

From topological entropy to ergodic theory for multivalued maps

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Abstract: The story of topological entropy for multivalued maps begins in 1976, when Mikhail Gromov, in his paper *On the Entropy of Holomorphic Maps*, proposed a possible definition—almost as a byproduct. This definition was independently rediscovered in 2017 by James Kelly and Tim Tennant. In my talk, I will outline how the development of entropy for multivalued maps has unfolded since then, and how I have contributed to this progression so far. Several cautious steps will be taken toward our ultimate goal, which can be described as the emergence of an *ergodic theory for multivalued maps*.

On Two Hypotheses About the Gao Beam

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Abstract: The nonlinear beam model introduced by Professor Gao in 1996 has become widely known as the Gao beam. In connection with this model, two key hypotheses concerning its mechanical behavior were formulated. The first states that the critical buckling load for the Gao beam is identical to that of the classical Euler–Bernoulli beam. The second claims that the Gao beam is generally stiffer than its Euler–Bernoulli counterpart. In this talk, we present a mathematical proof confirming the first hypothesis. However, the second hypothesis does not hold in general; this conclusion is supported by numerical simulations and data analysis. The results indicate that while the Gao beam exhibits comparable stability under buckling, its stiffness properties can vary depending on loading and boundary conditions.

A sufficient condition for existence of solutions for fractional ODE optimal control problem with convexity assumption

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Abstract: In this talk a sufficient condition for the existence of optimal solutions for the Lagrange problem with fractional Cauchy problem will be presented. The proof of the main result is based on the assumption of convexity of the value of a certain multivalued mapping, we use the Lower Closure Theorem and the Implicit Function Theorem for a multivalued mapping (Fillipov's lemma) .

Analyzing the Gao beam using data analysis

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Abstract: This contribution deals with the original static model of the Gao beam from 1996, see [1]. Although the nonlinearity contained in the basic equation is of a rather mild type, the properties resulting from it are not easily obtained by standard analytical procedures, which the authors of this contribution can unfortunately confirm. Therefore, it is appropriate to turn to less standard procedures, especially data analysis. We obtain large amounts of data using a large series of calculations and the analysis of these results is then performed partly using simple algorithms, i.e., say, using primitive AI (Artificial Intelligence), partly and finally using a much smarter and more sophisticated HI (Human Intelligence). In this way, it is possible to obtain, understandably at the cost of a considerable amount of work, quite interesting properties that would otherwise have remained undiscovered. The results presented here concern both convex and non-convex problems associated with the original Gao beam.

References

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Penalty method in the study of nonsmooth quasistatic frictional contact with locking materials

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Abstract: In the talk a class of time-dependent variational-hemivariational inequalities is studied. The existence and uniqueness of a solution are proved. Then, the convergence result for a penalized form of the problem is provided. Finally, the results are applied to a quasistatic frictional contact problem for locking materials. The contact is modeled with friction and a nonsmooth multivalued interface law. The law involves unilateral constraints and subdifferential conditions. The weak formulation of the problem leads to an elliptic variational-hemivariational inequality. Under suitable assumptions we obtain its weak unique solvability.

Finite Element Methods for fourth-order variational inequalities in elasticity

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Abstract: In this talk, we review some recent results established by the speaker and various collaborators of his.
The first part of the talk is devoted to establishing the convergence of a Finite Element Method for Koiter's model subjected to an obstacle. We establish this result by passing to an *ad hoc* mixed formulation to discretise the solution via Lagrange finite elements. This is joint work with Xiaoqin Shen and Xin Peng (Xi'an University of Technology, Peoples Republic of China).
The second part of the talk provides a counter-example asserting that there are fourth order obstacle problems whose solutions cannot be approximated by means of Lagrange triangles, even after passing to the corresponding mixed formulation. This result was obtained in collaboration with Tianyu Sun (Indiana University Bloomington, USA).

Existence of multiple solutions for certain classes of nonlinear anisotropic problems

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Abstract: The aim of this talk is to present existence and multiplicity results for certain classes of anisotropic Laplacian problems with Dirichlet boundary conditions. Our findings include two nontrivial weak solutions, proven both with and without relying on the Ambrosetti-Rabinowitz condition. Moreover, we highlight the existence of three solutions for a specific type of anisotropic Laplacian problem.

Convex optimization problems inspired by geotechnical stability analysis

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Abstract: This contribution is motivated by the *limit load*, *limit analysis* and *shear strength reduction* methods, which are commonly employed in geotechnical stability analysis or similar applications. The aim is to make these methods more approachable by introducing a unified framework based on abstract convex optimization and its parametric studies. We establish suitable assumptions on the abstract problems that capture the selected features of these methods and facilitate rigorous theoretical investigation. Further, we propose continuation techniques tailored to the resulting parametric problem formulations and show that the developed abstract framework could also be useful outside the domain of geotechnical stability analysis. The main results are illustrated with analytical and numerical examples. The numerical example deals with a 3D slope stability problem.
The research is supported by the European Union through the Operational Programme Jan Amos Komenský under project INODIN No. `CZ.02.01.01/00/23_020/0008487`.

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Multiple Solutions to Dirichlet Problems in Billiard Spaces

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Abstract: In [2], a Dirichlet problem in billiard spaces was considered. In the framework of impulsive differential equations, it can be written as a BVP

$$\begin{aligned} &x'' = f(t,x) \quad \text{if } x(t) \in \operatorname{int} K, \\ &x'(t+) = -x'(t-) \quad \text{if } x(t) \in \partial K, \\ &x(0) = A, \quad x(T) = B, \end{aligned}$$

where $T > 0$, $K = [0,R] \subset \mathbb{R}$, $R > 0$, f is a Carathéodory function on $[0,T] \times K$, $A, B \in \operatorname{int} K$. The main result of mentioned paper is the existence of solutions with prescribed number of impacts with the boundary. This was achieved using a method introduced in the same work—referred to here as a *tessellation technique*. We demonstrate that this technique can be extended in various directions. In the presentation, we will outline the tessellation technique and illustrate its application to more general problems, including those addressed in a recently submitted article [1].

References

[1] J. Andres and J. Tomeček. *Multiple solutions of vector Dirichlet problems to implicit differential equations and inclusions in billiard spaces.* submitted.

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Two Positive Solutions for an Elliptic Differential Inclusion driven by the Laplace Operator

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Abstract: We consider the following elliptic differential inclusion

$$\begin{cases} -\Delta u \in \lambda G(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{P_λ}$$

where Ω is a bounded open set in $\mathbb{R}^N$, $N \geq 3$, having a smooth boundary $\partial\Omega$, Δ is the classical Laplace operator, λ is a positive real parameter and $G : \mathbb{R} \to 2^{\mathbb{R}}$ is an upper semicontinuous set-valued mapping with compact convex values. We establish, under a subcritical growth condition on the function $\min G(t)$, the existence of a precise interval for the positive real parameter λ such that the problem (P_λ) admits two positive solutions. Moreover, an application to Dirichlet problems with equations having discontinuous nonlinearities is established, where it is worth noticing that the set of discontinuity points may also be uncountable.

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On Dimension Reduction and the Korn Inequality

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Abstract: In this talk, we explore the role of the Korn inequality in a range of problems related to dimension reduction, beginning with elasticity and extending to fluid dynamics. The Korn inequality plays a crucial role in understanding the behavior of stress tensors in thin domains, particularly as the diameter or thickness of these domains approaches zero. It serves as a key analytical tool for controlling the asymptotic behavior of the main physical quantities in such settings.