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Optimal control of systems governed by Stokes equations with non-smooth slip/leak conditions

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Abstract: In the first part of this contribution we present Stokes system equipped with several non-smooth slip and leak boundary conditions, recall their weak formulations and existence/uniqueness results. Special attention will be paid to the Stokes system with Tresca type leak condition which will be used in the subsequent part as the state problem. We derive its dual formulation in terms of three Lagrange multipliers releasing the divergence free condition, no-slip condition $u_t = 0$ on the leak part of the boundary, and regularizing the non-smooth leak term j, respectively. The algebraic counterpart of the dual formulation will be used as the state solver. The next part of the contribution deals with a class of optimal control problems in which the threshold leak bound g represents the control variable, Since the control-to-state mapping Φ is only Lipschitz continuous, sensitivity analysis being an integral part of any optimization problem has to be done using tools of nonsmooth analysis. We show how to get an "alternative" gradient information in the discrete case which will be then utilized in nonsmooth minimization methods. The last part of this contribution is devoted to computational aspects and several numerical experiments. We mention four approaches for solving this type of problems: a) fully nonsmooth, b) partially nonsmooth, c) fully smooth, d) gradient free global optimization approach. Our numerical experiments are based on b), c) and d) and the results will be compared.