Two Positive Solutions for an Elliptic Differential Inclusion driven by the Laplace Operator

Bruno Vassallo¹, Gabriele Bonanno, Valeria Morabito and Donal O'Regan

¹ University of Messina

bruno.vassallo@studenti.unime.it

Abstract: We consider the following elliptic differential inclusion

$$\begin{cases} -\Delta u \in \lambda G(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial \Omega, \end{cases}$$
 (P_{\lambda})

where Ω is a bounded open set in \mathbb{R}^N , $N \geq 3$, having a smooth boundary $\partial\Omega$, Δ is the classical Laplace operator, λ is a positive real parameter and $G : \mathbb{R} \to 2^{\mathbb{R}}$ is an upper semicontinuous set-valued mapping with compact convex values. We establish, under a subcritical growth condition on the function min G(t), the existence of a precise interval for the positive real parameter λ such that the problem (P_{λ}) admits two positive solutions. Moreover, an application to Dirichlet problems with equations having discontinuous non-linearities is established, where it is worth noticing that the set of discontinuity points may also be uncountable.

Acknowledgement: The first three authors are members of the Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni (GNAMPA) of the Istituto Nazionale di Alta Matematica (INdAM). This work is partially supported by PRIN 2022 "Nonlinear differential problems with applications to real phenomena (2022ZXZTN2)" and by INdAM - GNAMPA Project "Problemi Differenziali Non Lineari: Esistenza e Molteplicità di Soluzioni" - CUP E5324001950001.

References

 Bonanno G., Morabito V., O'Regan D., Vassallo B., Two Positive Solutions for Elliptic Differential Inclusions AppliedMath. 2024; 4(4): 1404-1417, https://doi.org/10. 3390/appliedmath4040074