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Multiple positive solutions for quasilinear nonlocal problem via topological, variational and set-valued methods

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Abstract: Our goal is to prove existence and multiplicity of positive solutions for the following problem

$$\begin{cases} -a\left(\int_\Omega u^q dx\right)\Delta_p u = f(x,u) & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{P}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary, and the driven operator is the classical p-Laplacian, i.e., $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ $\forall u \in W_0^{1,p}(\Omega)$. Moreover, we assume that a is a continuous functions, possible sign changing and $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function. The main novelty in our result is the absence of monotonicity conditions on the reaction term. Our result provides the existence of pairs of positive solutions for problem (P) ordered in L^q-norm. In particular, we combine sub-super solution and variational methods with truncation techniques to obtain positive solutions for a counterpart of the problem (P) with "frozen" non-local term. Next, to come back to solutions of our original problem, first, we prove the existence of the smallest solution for the "frozen" problem through a suitable set-valued map. Finally, we define a one-dimensional fixed point problem that leads to the existence, multiplicity and ordering of solutions of (P). We highlight that these nonlocal problems were introduced in 1945 by Carrier to describe the beams' deflection, but, in recent years, a wide interest in these equations has led many authors to obtain models in biology and engineering.

References

[1] P. Candito, G. Failla, L. Ganiński, R. Livrea *Multiple positive solutions for quasilinear nonlocal problem via topological, variational and set-valued methods*, (Preprint).